

<b>Grade Level/Course:</b> Algebra I
<b>Lesson/Unit Plan Name:</b> Factoring Quadratic Expressions
<b>Rationale/Lesson Abstract:</b> When factoring quadratic expressions, the guess and check method builds student's number sense and intuition and also reinforces their understanding and ability to multiply binomial expressions. One of the keys is to explicitly teach students how to guess effectively and efficiently.
<b>Timeframe:</b> 2 Days
<b>Common Core Standard(s):</b> A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.

**Instructional Resources/Materials:**

Handout to explicitly teach students to guess effectively and efficiently by recognizing which signs to use and how to use the value of the middle term to choose which factors to try first.

## Activity/Lesson:

### PRIOR KNOWLEDGE

When teaching binomial multiplication, be sure to:

- Teach using algebra tiles (if you plan to show factoring with algebra tiles)
- Teach using area model with a generic rectangle
- Teach using F.O.I.L.
- Teach using multiple distribution (helpful to reinforce distributive property)

### Begin by Building a Flowchart

- Initially, students can be taught to factor quadratic expressions step-by-step while building their own flowchart. A complete flowchart can be seen on the WCCUSD math department website under the Content Presentations tab. Click on the link for Algebra I and the flowchart is the last part of the presentation titled “Factoring: GCF, Multiple Methods for Factoring Trinomials, Difference of Squares”.
- Teach students to look for the GCF of the terms even though most of the time it will be 1 when students first start factoring. Teaching this from the beginning will instill a habit of looking to factor out the GCF first.
- If there is no GCF other than 1, students can then move to the next step.
- Once we actually start factoring out a GCF, it is important to teach students that they don't *have* to factor out the GCF first; only that this practice will make the process of factoring the quadratic expression simpler.

### Example 1:

Factor  $x^2 + 8x + 15$  completely.

Product	Sum
$\_\bullet\_\ = 15$	$\_\ + \_\ = 8$
$1 \bullet 15 = 15$	$1 + 15 = 16$
$3 \bullet 5 = 15$	$3 + 5 = 8$

#### Choral Response Questions

Expression, equation, or inequality?  
Expression.

What type of polynomial? *Trinomial*.

Is there a GCF other than 1? *No*.

#### Factoring by Grouping

##### Area Model

$$x^2 + 8x + 15$$

Based on our product/sum chart, we will use  $3x$  and  $5x$ .

$$= x^2 + 3x + 5x + 15$$

Fill in the area model.

$x^2$	$3x$
$5x$	$15$

Factor the GCF from each row and column.

$$x \quad +3$$

$x$	$x^2$	$3x$
$+5$	$5x$	$15$

$$= (x+5)(x+3)$$

##### Algebraically

$$x^2 + 8x + 15$$

$$= x^2 + 3x + 5x + 15$$

Group the first two terms and the last two terms.

$$= (x^2 + 3x) + (5x + 15)$$

Factor the GCF from each quantity.

$$= x(x+3) + 5(x+3)$$

$$= (x+5)(x+3)$$

#### Factoring by Guess & Check

##### Area Model

$$x^2 + 8x + 15$$

Fill in the first and last terms.

$x^2$	
	$15$

The first factors have to be  $x$  and  $x$ .

Strategically guess which factors of 15 to try.

$$x \quad +3$$

$x$	$x^2$	
$+5$		$15$

Fill in the remaining areas and check the diagonal like terms.

$$x \quad +3$$

$x$	$x^2$	$3x$
$+5$	$5x$	$15$

$$= (x+5)(x+3)$$

##### Algebraically

$$x^2 + 8x + 15$$

$$= (x \quad)(x \quad)$$

$$= (x + \quad)(x + \quad)$$

$$= (x+5)(x+3)$$

The First and Last terms are the same when we multiply the trial factors. What changes is the middle term. Thus, it is only necessary to check the Outer and Inner terms.

$$(x+5)(x+3)$$

$$3x + 5x = 8x \quad \checkmark$$

\*Note that when teaching area models, the area represents the trinomial and the dimensions represent the factors.

## Activity/Lesson continued

### You Try 1:

**Factor  $x^2 + 19x + 60$  completely. Use guess and check with an area model and algebraically.**

If needed students can still use a product/sum chart. The goal is for students to develop their number sense and intuition to factor quadratics without using a product/sum chart.

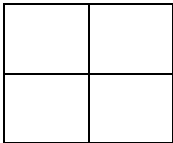
Product	Sum
$\_\_\_ \bullet \_\_\_ =$	$\_\_\_ + \_\_\_ =$

#### Choral Response Questions

Expression, equation, or inequality?

What type of polynomial?

Is there a GCF other than 1?

<u>Area Model using a Generic Rectangle</u>	<u>Algebraically</u>
$x^2 + 19x + 60$ <div style="text-align: center;">  </div>	$x^2 + 19x + 60$

Things to teach explicitly to help students guess effectively and efficiently:

**Example 2:** Factor the following expressions completely.

$x^2 + 4x + 3$	$x^2 - 4x + 3$	$x^2 + 2x - 3$	$x^2 - 2x - 3$
$= (x - 3)(x + 1)$	$= (x - 3)(x + 1)$	$= (x - 3)(x + 1)$	$= (x - 3)(x + 1)$

$$= (x+3)(x+1) \quad | \quad = (x-3)(x-1) \quad | \quad = (x+3)(x-1) \quad | \quad = (x-3)(x+1)$$

$x^2 + 8x + 7$	$x^2 - 8x + 7$	$x^2 + 6x - 7$	$x^2 - 6x - 7$
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Discuss if the third term is negative, one sign is negative and one sign is positive. The middle term determines the sign of the larger factor (note this is only always true when  $a = 1$ ).

## Activity/Lesson continued

### USING THE MIDDLE TERM TO GUESS MORE EFFECTIVELY

**You Try 3:** Factor the following expressions completely. Pay attention to the affect the middle term has on the factors.

$x^2 + 61x + 60$ $= (x \quad)(x \quad)$	$x^2 - 61x + 60$ $= (x \quad)(x \quad)$	$x^2 + 59x - 60$ $= (x \quad)(x \quad)$	$x^2 - 59x - 60$ $= (x \quad)(x \quad)$
$x^2 + 32x + 60$ $= (x \quad)(x \quad)$	$x^2 - 32x + 60$ $= (x \quad)(x \quad)$	$x^2 + 28x - 60$ $= (x \quad)(x \quad)$	$x^2 - 28x - 60$ $= (x \quad)(x \quad)$
$x^2 + 23x + 60$ $= (x \quad)(x \quad)$	$x^2 - 23x + 60$ $= (x \quad)(x \quad)$	$x^2 + 17x - 60$ $= (x \quad)(x \quad)$	$x^2 - 17x - 60$ $= (x \quad)(x \quad)$
$x^2 + 19x + 60$ $= (x \quad)(x \quad)$	$x^2 - 19x + 60$ $= (x \quad)(x \quad)$	$x^2 + 11x - 60$ $= (x \quad)(x \quad)$	$x^2 - 11x - 60$ $= (x \quad)(x \quad)$
$x^2 + 17x + 60$ $= (x \quad)(x \quad)$	$x^2 - 17x + 60$ $= (x \quad)(x \quad)$	$x^2 + 7x - 60$ $= (x \quad)(x \quad)$	$x^2 - 7x - 60$ $= (x \quad)(x \quad)$
$x^2 + 16x + 60$ $= (x \quad)(x \quad)$	$x^2 - 16x + 60$ $= (x \quad)(x \quad)$	$x^2 + 4x - 60$ $= (x \quad)(x \quad)$	$x^2 - 4x - 60$ $= (x \quad)(x \quad)$

What do you notice about how the middle term of the trinomial affects what factors you try first?

- Discuss the closer the middle term is to zero in the trinomial, the closer the factors will be in the binomials.
- Also, discuss the relationship between the sign of larger factor in the binomial and the sign of the middle term from the trinomial for the last two columns. When the last term of the trinomial is negative, the sign of the larger factor in the binomials should match the sign of the sum (note again this is only always true when  $a = 1$ ).
- By choosing a trinomial whose first term is  $x^2$ , and whose last term is  $\pm 60$ , it will help students understand how to make better guesses about which factors to use. Also, by choosing the number 60 which has 12 whole number factors, and varying its sign, students will have the ability to factor 24 similar trinomials.

## Assessment

Factor the following expression using guess and check with an area model and algebraically.

$$x^2 + 5x - 24$$

### Guess & Check

**Area Model**

**Algebraically**

## HANDOUT

Factor the following 24 expressions. Be ready to discuss how we know the signs of the binomials and how the middle term of the trinomial helps you decide which factors of 60 to use.

$x^2 + 61x + 60$ $= (x \quad)(x \quad)$	$x^2 - 61x + 60$ $= (x \quad)(x \quad)$	$x^2 + 59x - 60$ $= (x \quad)(x \quad)$	$x^2 - 59x - 60$ $= (x \quad)(x \quad)$
$x^2 + 32x + 60$ $= (x \quad)(x \quad)$	$x^2 - 32x + 60$ $= (x \quad)(x \quad)$	$x^2 + 28x - 60$ $= (x \quad)(x \quad)$	$x^2 - 28x - 60$ $= (x \quad)(x \quad)$
$x^2 + 23x + 60$ $= (x \quad)(x \quad)$	$x^2 - 23x + 60$ $= (x \quad)(x \quad)$	$x^2 + 17x - 60$ $= (x \quad)(x \quad)$	$x^2 - 17x - 60$ $= (x \quad)(x \quad)$
$x^2 + 19x + 60$ $= (x \quad)(x \quad)$	$x^2 - 19x + 60$ $= (x \quad)(x \quad)$	$x^2 + 11x - 60$ $= (x \quad)(x \quad)$	$x^2 - 11x - 60$ $= (x \quad)(x \quad)$
$x^2 + 17x + 60$ $= (x \quad)(x \quad)$	$x^2 - 17x + 60$ $= (x \quad)(x \quad)$	$x^2 + 7x - 60$ $= (x \quad)(x \quad)$	$x^2 - 7x - 60$ $= (x \quad)(x \quad)$
$x^2 + 16x + 60$ $= (x \quad)(x \quad)$	$x^2 - 16x + 60$ $= (x \quad)(x \quad)$	$x^2 + 4x - 60$ $= (x \quad)(x \quad)$	$x^2 - 4x - 60$ $= (x \quad)(x \quad)$